Developments in Pre-service Teachers' Mathematics for Teaching of Fractions

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The mathematics required for teaching is increasingly becoming an important issue for research (Ball, Thames & Phelps, 2008). This study examines the development and quality of the mathematical content knowledge of a cohort of pre-service primary teachers. We commenced the study of the impact of a Model-Based Teaching and Learning (MBTL) approach on the development of pre-service teachers' conceptual knowledge in the domain of fractions (Forrester & Chinnappan, 2011). In this present study we found further evidence for the robustness of MBTL as an effective instructional strategy in promoting conceptual knowledge.

Issue and Context

The quality of teacher knowledge in relation to children's learning outcomes in mathematics (Ball, Hill, & Bass, 2005) has highlighted the need to research and monitor the developing knowledge of mathematics teachers who are in practice and those who are in training. Towards this end the broad area of inquiry focusing on *Mathematics for Teaching* (Ball, Thames, & Phelps, 2008) is significant. Interest in the types of knowledge necessary for teaching has its roots in the work of Shulman (1987) who conceptualised that teaching (in any domain) needs to be supported by subject-matter knowledge that in turn has to be reshaped into pedagogical content knowledge (PCK).

Ball and her associates (Ball, 2003; Ball, et al., 2008), building on Shulman's work, have identified several types of content knowledge necessary for teaching, two of which are particularly relevant to this research, namely, Common Content Knowledge (CCK) and Specialised Content Knowledge (SCK). CCK is the content knowledge used in a range of settings whereas SCK is the content knowledge "unique to teaching" (Ball, et al., 2008, p. 400).

Our research in this area was motivated by the need to better understand the quality SCK of primary teachers of mathematics who are enrolled in a pre-service program. Our previous research was focused on comparing the procedural and conceptual knowledge of fractions of a cohort of Pre-Service Teachers (PSTs) at the end of their first year of undergraduate studies (Forrester & Chinnappan, 2010). This cohort will be referred to as Cohort 1. The results of this analysis demonstrated the dominance of procedural over conceptual knowledge in this group, with almost four times the number of pre-service teachers activating procedural knowledge in comparison to those that demonstrated conceptual knowledge in their solution attempts. About one fifth of responses evidenced neither procedural nor conceptual knowledge.

While both knowledge categories are important, the dominance of procedural over conceptual knowledge would seem to be unhealthy for classroom practice, as teachers will have to support the development of both across all strands of primary mathematics, including fractions. This line of reasoning motivated us to modify our teaching strategies

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with the view to enhancing the conceptual component of our pre-service teachers' knowledge of fractions. The following year, we sought to address this lack of conceptual understanding by adopting a Model-Based Teaching and Learning (MBTL) strategy with a second cohort of first year PSTs (Cohort 2). The results of this teaching intervention produced a significant improvement in the balance of the two knowledge categories with about 65% of the students who received MBTL demonstrating conceptual knowledge (Forrester & Chinnappan, 2011).

In the present study we report the results of our continuing work in the effectiveness of MBTL in fostering the construction and development of conceptual knowledge in the domain of fractions by examining changes in the knowledge of Cohort 1 whose knowledge was shown to be predominantly procedural (Forrester & Chinnappan, 2010), as they completed their third year mathematics content and pedagogy subject.

Objective

The aim of this study was to ascertain the impact of a MBTL approach on the development of procedural and conceptual knowledge in the domain of fractions among a cohort of pre-service teachers (PSTs) as they progressed from the first to third year of their undergraduate program. The following research question guided the study:

1. Does a MBTL have an impact on the development of pre-service primary teachers' conceptual and procedural knowledge of fractions?
2. Does a MBTL approach have an impact on the development of pre-service primary teachers' procedural knowledge of fractions?

Relevant Literature

Model Based Teaching and Learning (MBTL)

The focus of our teaching approach was on models and modelling of fractions. By focusing on models, we adopted the view that PSTs construct and reason with entities called mental models (Johnson-Laird, 1983). These are internal representations of the targeted phenomena (fractions) that are being modelled. Such models can be given external representations by the learner in the form of actions, speech and visuals. Our teaching approach was driven by the need to assist learners to engage in developing and expressing mental models. This view is aligned with that of Gobert and Buckley (2000) who argued that model-based teaching 'is any implementation that brings together information resources, learning activities, and instructional strategies intended to facilitate mental model-building both in individuals and among groups of learners' (p. 892). The strategies for MBTL align with the principles suggested by Gobert and Buckley (2000) which describes model-based teaching as "any implementation that brings together information resources, learning activities, and instructional strategies intended to facilitate mental model-building" (p. 892).

Procedural versus Conceptual Knowledge

Procedural knowledge involves understanding the rules and routines of mathematics while conceptual knowledge involves an understanding of mathematical relationships. The relationship between procedural and conceptual knowledge, and the dependency of one on the other, continues to be a legitimate concern for mathematics teachers and researchers alike. Schneider and Stern (2010), in examining potential interconnections between the two, suggested that teaching and learning research needs to examine their parallel developments.

Within the context of primary mathematics, in particular fractions, Mack (2001) suggests that children's use of strategies for representing and solving fractions problems are based on both these knowledge strands. The relationships amongst and the relative roles of these two main dimensions of knowledge, relevant to decoding and solving fractions problems, needs further clarification if we are to better inform teachers about knowledge underlying teaching.

Method

Participants

One hundred and twenty PSTs (14 males and 106 females) participated in the present study. They were enrolled in a third year compulsory subject, which is generally completed in the third year of a four year Bachelor of Primary Education degree. They were completing the second core mathematics content and pedagogy subject in their degree, having completed the first core subject in the first year of their degree. Their responses to fraction exam questions in the first core subject were reported previously (Forrester & Chinnappan, 2010). Prior to entry into the degree course, the participants had a range of mathematical backgrounds.

Model Based Teaching and Learning

The MBTL approach used in this project evolved from the use of animated PowerPoint models provided in lectures and as support materials to develop PSTs' conceptual understanding of fraction concepts and operations. While these models were used to explain fraction concepts and operations, students were not actively engaged in the modelling process and only procedures were examined to assess understanding.

In the 2009 rendition of the first year core subject, efforts were made to focus more specifically on developing PSTs' conceptual understanding of mathematical concepts. The assessment of PSTs required them to demonstrate their knowledge of fraction concepts and operations by providing both a calculation and a model. Exam responses were analysed in terms of the evidence of procedural and/or conceptual understanding and many PSTs evidenced poor conceptual understanding of fraction concepts (See Table 2) with nearly 20% of students evidencing neither forms of understanding, (Forrester & Chinnappan, 2010).

These results led to the introduction of MBTL being introduced into the core mathematics subjects, where the models previously made by the lecturer in demonstrating and explaining mathematical concepts, were also constructed and explained by PSTs to develop their own conceptual understandings. Utilising the representational-reasoning model of understanding theorised in the work of Barmby, Harries, Higgins and Suggate (2009), which maintains that robust mathematical understanding is demonstrated when learners can construct and utilise multiple representations of mathematical ideas and can justify the relationships among representations, we focused on enabling our students to develop and explain models of fraction concepts and operations as well as utilising appropriate algorithms to solve fraction problems.

Tasks

The following fraction problems were provided as questions in the final examination of the core mathematics content and pedagogy subjects in the degree. Tasks 1a and 1b involved subtraction while Tasks 2a and 2b involved multiplication. Structurally, 1b is similar to 1a; likewise with Tasks 2a and 2b. Task 1a and 2a were completed in 2009 whereas their variants (1b and 2b) were attempted by the same PSTs in 2011. These particular tasks were chosen to examine students' mathematics content knowledge in terms of their conceptual and procedural knowledge of fractions and fraction algorithms.

Task 1a: $1\frac{2}{5}-\frac{5}{6}$

Task 1b: $1\frac{3}{8}-\frac{3}{4}$

Task 2a: $\frac{1}{4}\times\frac{2}{3}$

Task 2b: $\frac{1}{3}\times\frac{3}{5}$

In attempting to solve the problems, PSTs were instructed to i) complete these calculations and ii) model the operations.

Coding Scheme

Students' responses to each of the two problems were analysed in terms of their demonstration of conceptual and procedural knowledge, and coded using a five code scale (see Table 1). The development of the coding scheme was guided by the theoretical framework of Barmby, Harries, Higgins and Suggate (2009) and Goldin's (2008) analysis of problem representations.

Table 1
Coding Scheme

0	No evidence of procedural or conceptual understanding
1	Procedural. Correct algorithm and/or model only of fractions
2	Procedural/conceptual. Correct algorithm and model demonstrates more than a procedural understanding of a concept involved in fraction operations
3	Conceptual. Correct algorithm, model of concept evident
4	Conceptual with explicit reasoning. Correct algorithm, model supported with language

Results

The MBTL approach that underpinned our teaching was aimed at examining changes procedural and conceptual knowledge of PSTs. Quantitative data analyses were conducted with the aid of SPSS version 18. Table 2 shows the percentage figures for the respective codes for the subtraction and multiplication problems respectively for 2009 and 2011, for the entire cohort of participants.

We note a number of patterns that are relevant to our two key research questions about the impact of MBTL on the development of procedural and conceptual knowledge. Firstly, in 2009, for both the problem types, there are higher percentages with codes 0, 1, and 2 (predominantly procedural knowledge) in comparison to codes 3 and 4 (supporting conceptual knowledge). We see the reverse in 2011, for both subtraction and multiplication problems. We thus have preliminary support for our expectation that a MBTL approach has induced among our PSTs the development of conceptual knowledge of fractions.

Table 2
Percentage Figures for the Codes

Code	Subtraction Problem		Multiplication Problem	
	2009	2011	2009	2011
	%	%	%	%
0	17.8	7.0	18.7	7.0
1	53.3	5.2	58.9	4.3
2	17.8	20.0	9.3	17.4
3	11.2	49.6	10.3	43.5
4	0	18.3	2.8	27.8

The above pattern is also reflected in particular cases of modelling as evidenced by solutions provided PST1 and PST 2.

Case of one PST1

In both the responses PST1 produced correct solutions by activating an appropriate algorithm (Figures 1 and 2). However, the model that was constructed in 2009 indicates misconceptions regarding the relationship between parts and wholes, and the notion of equivalence (Figure 1). These misconceptions seem to have been corrected in the model that was constructed in her 2011 solution attempt (Figure 2).

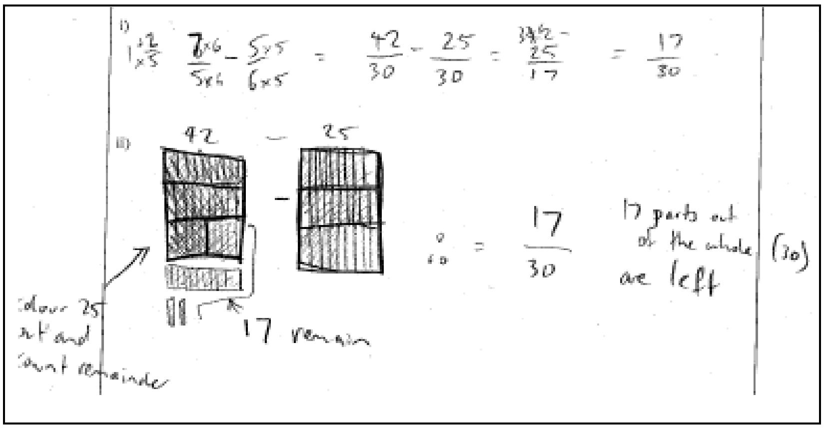

Figure 1. PST1's responses to subtraction task in 2009

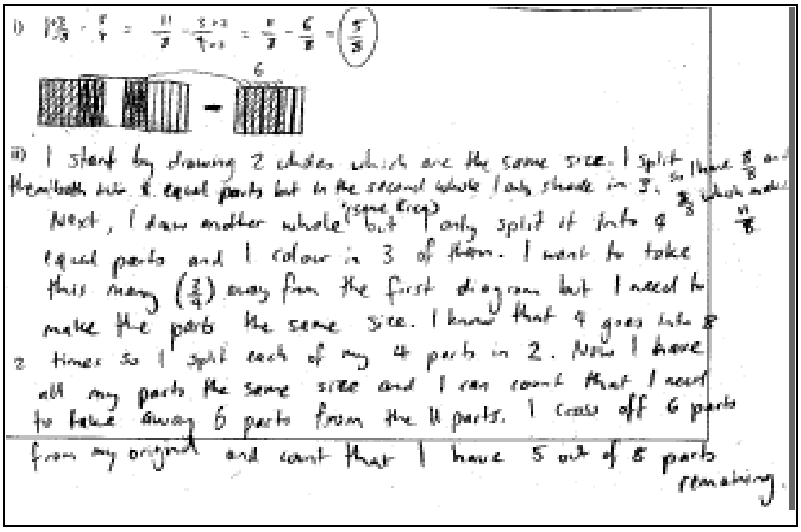

Figure 2. PST1's responses to subtraction task in 2011

PST2 also produced correct solutions to the multiplication problems in both the 2009 and 2011 solution attempts by drawing on and executing a correct algorithm (Figures 3 and 4). We note the model produced in 2011 (Figure 4) engages with the concept of multiplying by a fraction rather than simply representing the procedure diagrammatically. The 2009 model did not engage substantively with the notions of equivalence or the relationship of parts to the whole, whereas the 2011 model demonstrates an understanding of these concepts (Figures 3 and 4). These responses illustrate the types of improvement evident among many PSTs in this cohort following our intervention with a MBTL approach.

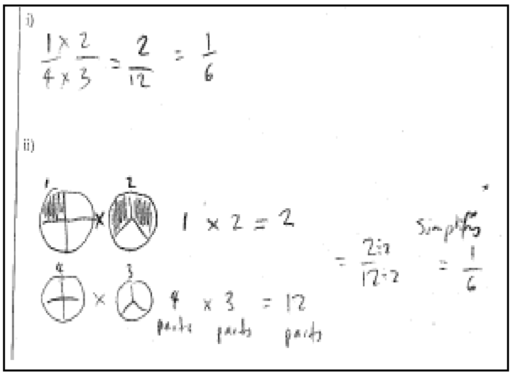

Figure 3. PST2's responses to multiplication task in 2009

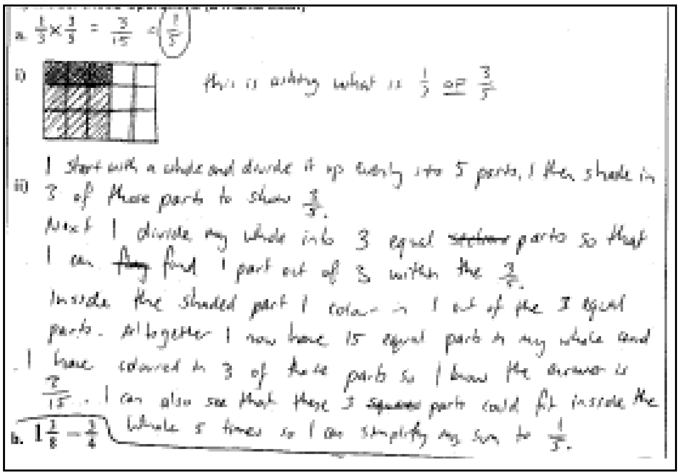

Figure 4. PST2's responses to multiplication task in 2011

Discussion

Ball, Hill and Bass (2005), in their articulation of *Mathematical Knowledge for Teaching*, suggested that teachers need to construct and deconstruct content knowledge in particular forms such that that knowledge is germane to learning. This cluster of knowledge was referred to as Specialised Content Knowledge (SCK). In our study we have attempted to unpack this knowledge further along conceptual and procedural lines. In so doing we argue that conceptual knowledge may subsume procedural knowledge and indeed contribute to a better understanding of related procedural knowledge, and it is important to capture and support both strands of knowledge if we are to develop metrics of SCK both quantitatively and qualitatively. Is conceptual knowledge better than procedural knowledge for practice? We suggest that there has to be a balance and that teachers' SCK ought to exhibit both these characteristics.

The research questions were concerned with the impact a teaching approach based on modelling would have on the development of PSTs' procedural and conceptual knowledge of fractions. The results here suggest that, while there was no tangible effect on procedural knowledge, our teaching had a positive affect on pre-service teachers' conceptual knowledge. The question is how did MBTL influence the conceptual knowledge of the PSTs? Two key facets of MBTL were a) the representation of the fraction symbols into equivalent visual forms and b) the interpretation of the operations in terms of the visuals. We suggest that these separation and subsequent integration was instrumental in developing conceptual knowledge. For example, Task 2a was interpreted as 'one quarter of two thirds'. Such an interpretation was also better aligned with the visual representations of both the fractions.

The positive effect of MBTL on conceptual knowledge may also have a similar impact on the confidence of our PSTs in teaching fractions. This is a hypothesis on our part and is based on the premise that less confident teachers tend to draw on mathematical knowledge that is more procedural in nature. Murphy (2011), for instance, found support for the above contention in her study of prospective teachers' knowledge underlying the concept of area.

There is an on-going debate within the research community about the relations between procedural and conceptual knowledge, regardless of the domain in which these are studies (Rittle-Johnson, Siegler, & Alibali, 2001; Schneider & Stern, 2010). In the present study, we have added to this debate by not isolating the two strands of knowledge in the domain of fractions but also showing the need for the two to be tightly connected so that we can support better learning among children.

One limitation of the present study is that it was not conducted within an experimental design that involved treatment and control conditions. Thus we are guarded in making claims about the efficacy of MBTL. We suggest that future studies use an experimental design with pre- and post-tests to better measure the effect of MBTL on the development of procedural and conceptual knowledge.

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